Zadar • Croatia • June 2 – 3, 2026

1st German – Croatian Workshop on Mechanics

Book of Abstracts

Editors
Marko Čanađija • Laura De Lorenzis • Lovre Krstulović-Opara
Tomislav Lesičar • Jörn Mosler • Jörg Schröder

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Preface

The cooperation between members of the German Association of Applied Mathematics and Mechanics (GAMM) and the Croatian Society of Mechanics (CSM) has existed for a long time. Their collaboration follows the usual patterns of research communities: joint research, co-authored papers, collaborative projects, research visits, and more. These activities have also led to the now traditional invitations of distinguished professors from Germany to deliver plenary lectures at the International Congress of the Croatian Society of Mechanics, which has been held as a triennial event since 1994. It is fair to say that these lectures have fostered relationships that extend beyond purely professional interactions, evolving into long-term friendships. We refrain from listing all the professors in this circle here to avoid inadvertently omitting anyone.

A natural outcome of this collaboration was the desire to elevate it to a higher level. In December 2022, Marko Čanađija, who was to become President of CSM for the 2023 - 2025 term, and Jörg Schröder, then Vice-President of GAMM, conceived the idea of signing a formal cooperation agreement between the two societies. The agreement was ultimately signed in February 2024 by Marko Čanađija and Karsten Urban, Presidents of CSM and GAMM, respectively.

With the agreement in place, the next question naturally arose: how can this cooperation be put into practice? Initial ideas included GAMM supporting CSM in reviewing candidates for the newly established Croatian Society of Mechanics Award for Young Scientists, while CSM would take on the organization of the prestigious GAMM Annual Meeting, scheduled to be held in Zagreb, Croatia, in 2029. The main organizer is the University of Zagreb, while the co-organizers are CSM and the Croatian Mathematical Society.

The first step toward the 1st German-Croatian Workshop on Mechanics was taken at the University of Duisburg-Essen in November 2024, during the celebration of Jörg's 60th birthday. The event, in fact a mini-conference dedicated to his scientific achievements, brought together many of his colleagues and friends, including Marko. After the lectures, in a relaxed and festive atmosphere, Jörg turned to Marko and remarked: "Now that we have a cooperation agreement, how can we cooperate?" As several presentations had highlighted bilateral workshops between different countries, the natural proposal emerged to organize a German-Croatian workshop on mechanics. The guiding idea was to bring together researchers from both countries and foster new collaborations.

The next step was to assemble organizing teams from both sides. From the German side, the natural choice was Jörn Mosler, as Jörn and Marko have collaborated for more than 15 years. Laura De Lorenzis, also a frequent plenary speaker at the International Congress of the Croatian Society of Mechanics, joined to complete the German team. On the Croatian side, Lovre Krstulović-Opara and Tomislav Lesičar became part of the organizing committee.

Besides registration fees, the workshop is financially supported and jointly organized by CSM, GAMM, and the University of Zadar, with the latter also kindly providing the lecture venue at its beautiful seaside location in Zadar, Croatia. The event is held under the auspices of the Kroatischer

Humboldtianer-Klub, made possible through the efforts of the long-standing ambassador of Croatian - German cooperation in mechanics, academician Jurica Sorić.

After all the efforts invested by both sides, the workshop will bring together 34 participants. Plenary lectures will be delivered by distinguished professors Peter Wriggers and Gordan Jelenić, while 25 additional speakers have been invited to present keynote lectures covering a wide range of cutting-edge research in solid and structural mechanics.

We wish you a successful workshop and fruitful cooperation in the future.

Marko Čanađija, Laura De Lorenzis, Lovre Krstulović-Opara,

Tomislav Lesičar, Jörn Mosler and Jörg Schröder

Zadar, June 2026

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Session Chairs: Kevin Linka and Mijo Nikolić

Plenary lectures

1st German-Croatian Workshop on Mechanics
June 2 – 3, 2026
Zadar, Croatia

Third Medium Contact: A New Framework and Recent Advances

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The third medium contact approach has been successfully employed in structural applications and extended to various optimization problems, see [1], [2] and [3]. This discretization technique replaces classical contact formulations and algorithms by introducing a compliant interfacial layer - referred to as the third medium - between the contacting bodies. Unlike traditional contact methods, this formulation naturally accommodates finite deformations at the interface. As the two bodies approach each other, the third medium undergoes compression and acts as a barrier, preventing interpenetration and transmitting contact forces in a smooth and stable manner.

The approach is also applied to coupled problems, with focus on thermo-mechanical analysis. Then heat conduction must be incorporated into the model, which typically requires specialized interface laws when using classical contact formulations. These laws aim to capture the complex thermal behavior at the contact interface, including discontinuities and varying conductance. In contrast, the third medium approach offers the advantage to account for the interface behavior without the need for additional interface conditions. This includes the gradual heat transfer through the surrounding gas when the bodies are near each other, as well as the localized heat conduction that occurs upon physical contact. As a result, the third medium naturally captures both non-contact and contact-phase thermal conduction within a unified framework.

In this talk, we discuss the different approaches that can be applied within third media contact discretization schemes for linear and quadratic ansatz functions. The discretization is carried out using finite and virtual elements [4] and then applied to mechanical and thermo-mechanical contact problems in two- and three-dimensions. The presentation includes comparison of different discretization schemes and algorithms in the light of robustness, efficiency and accuracy. A more complex finite deformation contact problem using self-stabilized virtual elements is shown below..

Figure 1 depicts a 2D plane-strain contact problem between a deformable annulus and a grooved foundation. The lower body is a rectangular block of width 10 and height 4, containing a centrally located semi-circular cavity. A circular ring is positioned above the cavity. The ring has outer radius $r_2 = 2.0$ and inner radius $r_1 = 1.7$, so that it can deform into contact with the concave surface. A vertical prescribed displacement $u_y = 4.5$ is applied at the top point of the ring, pushing it downward into the cavity. The normal gap between the ring and the cavity is denoted by g_n.

This benchmark is intended to study finite-deformation contact, large local curvature effects, and illustrates the ability of the virtual element method for non-matching discretisations in the contact zone.

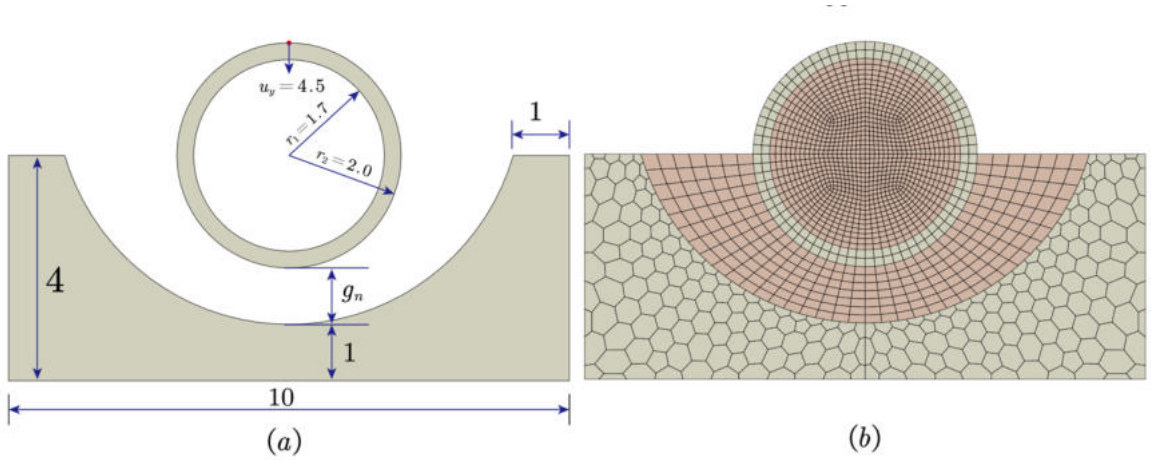

Fig. 1 (a) System and (b) mesh using virtual elements

Figure 2 illustrates the complete contact process by showing the deformed configurations at different applied vertical displacements u_y.

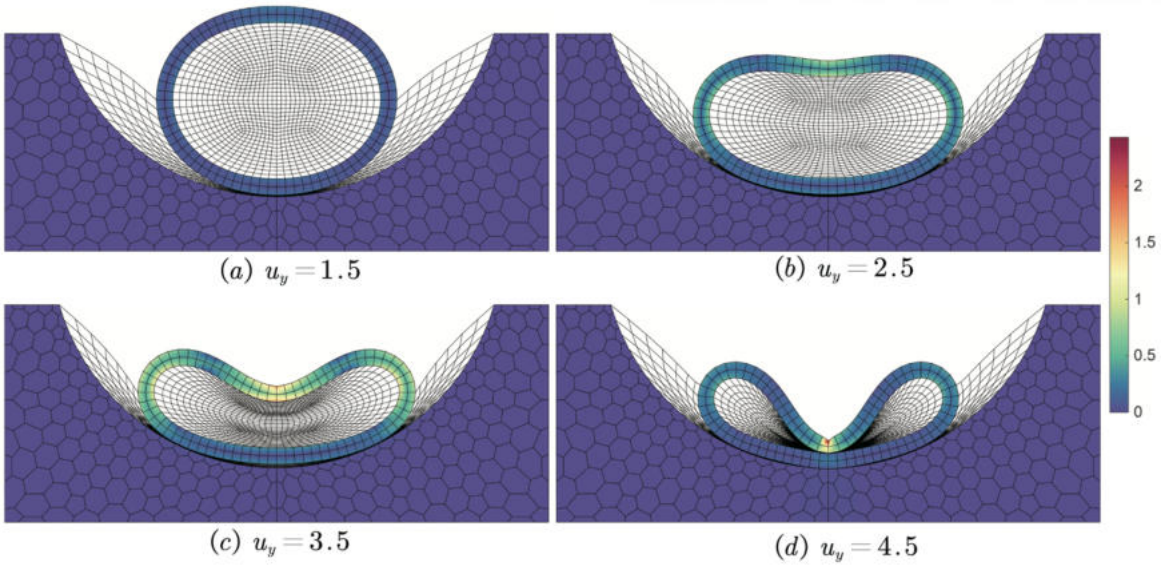

Fig. 2 Deformed states and von Mises stresses for different applied displacements.

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On Some Open Questions in Linear and Non-Linear Cosserats' Elasticity

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1. Introduction

Cosserats' continuum theory provides a model for describing certain experimentally observed phenomena unrecognised by Cauchy's theory. These phenomena reveal some less known effects when the methods of classical elasticity are rigorously extended to Cosserats' theory.

2. Theoretical justification for a negative size effect in Cosserats' elasticity

The size effect in Cosserats' theory is caused by the extra material parameters of the theory, which are limited by the condition of positive strain-energy density implying that smaller specimens may be only stiffer than larger ones. When the requirement of the positive strain-energy *over a representative volume* is used instead, the range of admissible parameter values is extended, giving a rigorous theoretical justification for the negative size effect [1].

3. Power-conjugate Cosserats' stress/couple-stress and strain/curvature tensors

For the family of Seth-Hill's strain tensors in Cauchy's elasticity, a unique family of power-conjugate stress tensors is obtained. In Cosserats' theory both of those tensors are non-symmetric and establishing power-conjugate tensor pairs is more complicated and highly unlikely for an arbitrary Seth's exponent. Often enough, however, this may be achieved and generalised to derive a corresponding family of power-conjugate Cosserats' curvature and couple-stress tensors [2].

4. Pure 2D bending in non-linear Cosserats' elasticity

This problem has been analysed for the semi-linear material in [3] exposing a balancing act between Cosserats' stiffening and non-linear softening challenging the standard linear pure-bending size-effect estimate. To generalise this solution to the whole class of Hill's materials along the above lines a simple yet fully consistent solution is derived.

5. Conclusions

The ability of Cosserats' continuum theory to rigorously account for a negative size effect and a general approach at developing power-conjugate stress and couple-stress tensors to a family of Seth-like strain and curvature tensors have been presented. This approach provides a surprisingly simple generalisation of the solution of 2D pure bending in classical hyperelasticity to Cosserats' materials.

Acknowledgement - This research is supported by the Croatian Science Foundation (HRZZ-IP-2024-05-9904).

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Keynote lectures

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A novel neural network for isotropic polyconvex hyperelasticity with guaranteed universal approximation

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1 Motivation

Accurate modeling of isotropic hyperelastic materials requires constitutive laws that satisfy fundamental principles such as frame-indifference, isotropy, and polyconvexity while remaining sufficiently expressive [1, 2, 3]. Existing neural network approaches typically enforce these constraints via sufficient conditions that are not necessary and therefore limit approximation capabilities. Ensuring all constraints while retaining universal approximation hence remains a key challenge.

2 Methodology

This work introduces a novel class of neural network models based on input convex networks formulated in terms of the signed singular values of the deformation gradient. Building on necessary and sufficient conditions for frame-indifferent, isotropic polyconvex energies, the approach enables the construction of Convex Signed Singular Value Neural Networks (CSSV-NNs [4]), including extensions tailored to incompressible materials (inc-CSSV-NNs [5]). In contrast to prior methods relying solely on sufficient criteria, the proposed formulation allows a complete characterization of admissible energies.

3 Results and Conclusion

The proposed framework can approximate any frame-indifferent, isotropic polyconvex energy given sufficient network capacity. A universal approximation theorem is established for both compressible and incompressible cases. Numerical experiments demonstrate accurate recovery of classical models (Neo–Hookean, Mooney–Rivlin, Gent, Arruda–Boyce) and agreement with Treloar's experimental data. Moreover, the approach outperforms models based on purely sufficient criteria, such as Ball's polyconvexity condition and invariants of the right Cauchy–Green tensor. Overall, it provides a theoretically rigorous and computationally efficient approach to hyperelastic material modeling, combining strict physical consistency with full approximation power.

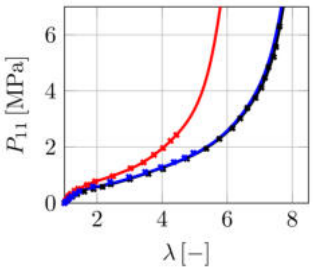

Figure 1. Stress over stretch of the CSSV-NN vs. Treloar data (solid lines vs. crosses) for uniaxial/equibiaxial/shear loading (black/red/blue).

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Computational Modelling of Heterogeneous Materials: Multiscale Methods, Constitutive Modelling and Data-Driven Approaches

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The mechanical behaviour of materials is largely influenced by their microstructure, which consists of various constituents. To take into account the influence of the microscale of materials, the last few decades have been characterized by an intense development of multiscale methods [1]. After solving the boundary value problem at the microscale, the results are homogenized. The whole process leads to long computational time and high computational costs. Among novel multiscale algorithms and homogenization procedures lowering the computational time, a data-driven, self-consistent clustering algorithm shows very promising behaviour. It is based on the data compression using *k*-means clustering, leading to a significant reduction in the number of degrees of freedom at the microscale [2].

Fracture is one of the most commonly encountered failure modes in modern engineering materials and structures. Today, fracture mechanics is a standard part of numerical methods, mostly FEM. Herein, diffusive approaches rely on the introduction of a scalar damage variable that controls the degree of material damage, as in the Phase-Field (PF) fracture formulation [3]. What makes this approach particularly attractive is its ability to elegantly simulate complicated fracture processes, without the need for additional ad-hoc criteria.

The description of the mechanical behaviour of materials has traditionally been based on analytical expressions derived from numerous experimental tests. Given the growing demand for the development of new materials with complex microstructures, it is essential to apply new and more efficient methods for describing the constitutive behaviour of materials. Machine learning methods based on neural networks have become a popular approach in recent years. Recent advances in artificial intelligence have demonstrated the potential of neural networks as surrogate models for modelling history-dependent homogenised constitutive responses [4].

Acknowledgment

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Towards operator learning for computational solid mechanics

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1. Introduction

Neural operators have emerged as powerful surrogates for PDEs, but their application to solid mechanics is often limited by difficulties in handling complex and highly variable boundary conditions, and their use in challenging problems such as damage and fracture remains at an early stage.

2. From physics-informed neural networks to neural operators for computational solid mechanics

This presentation first introduces physics-informed neural networks [1] and neural-operator-based surrogates [2,3] for accelerating brittle fracture simulations based on the phase-field approach, accurately predicting crack nucleation, propagation, and branching. It then illustrates a general framework for conditioning neural operators on arbitrary boundary functions through learned boundary-to-domain extensions [4], enabling standard architectures to robustly incorporate rich boundary information without PDE-specific modifications and achieving state-of-the-art accuracy, robustness to noise, and strong transferability across linear and nonlinear benchmark problems.

3. Conclusions

Together, these advances demonstrate how modern neural operator methodologies hold promise to deliver accurate, efficient, and scalable surrogate models for challenging solid mechanics applications.

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Bayesian Stochastic Identification of the Discrete Model Parameters in Multi-Physics and Fracture Problems

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1. Introduction

This work presents the Bayesian probabilistic identification of parameters in discrete lattice models, to provide a link between discrete and continuum representations. It considers the three main applications: first, a plane stress formulation for calibrating lattice model parameters to accurately reproduce two-dimensional mechanical behavior, including correct lateral deformations and Poisson effects. Second, a three-dimensional multiphysics extension is presented through a thermo-hydro-mechanical framework for soil applications for the identification of coupled thermal, hydraulic, and mechanical parameters. Finally, the probabilistic identification is extended to nonlinear fracture regimes where failure parameters are identified on a dog-bone specimen from uniaxial test responses.

2. The methodology and outputs

The methodology is based on a Bayesian framework for parameter identification [1]. Unknown and uncertain model parameters are treated as random variables and updated using observed or synthetic data. The posterior distributions are obtained through the application of Bayes' theorem, while the exploration of the parameter space is performed using Markov Chain Monte Carlo (MCMC) methods. The reduction of the computational cost associated with vast number of required computations of the discrete lattice model is achieved by using proxy models replacing discrete element simulations. For the identification of elastic parameters, such as elastic correction coefficients of the model [2-3], a generalized Polynomial Chaos (gPC) surrogate is used, while for nonlinear fracture problems [4], the parameter space is extended to include mechanical properties such as individual element tensile strength, shear strength, and corresponding tensile and shear fracture energies. Surrogate modeling in this case is performed using neural networks, which provide more capable approach to capture highly nonlinear relationships between input parameters and structural response. The discrete lattice model is based on a Voronoi tessellation with one-dimensional beam elements connecting neighboring cells [2]. These lattice elements are capable of carrying coupled mechanical, thermal, and pore pressure fields, enabling the simulation of multiphysics behavior within the same framework [3].

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The Power of Auxetics: Designing Metamaterials from the Inside Out

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1 Abstract

Auxetic structures form a distinct subgroup of metamaterials and possess unique characteristics, making them of great interest for aerospace engineering. Noteworthy properties such as the negative Poisson ratios, high resistance to penetration, high energy dissipation as well as energy absorption capabilities, depend mainly on the geometry of the underlaying auxetic unit cell. Within this talk the transition from investigating a single or patches of a few unit cells to practice relevant structures is highlighted. On the unit-cell level influences of the geometry as well as the position of a unit cell on Poisson ration and energy absorption are studied both, experimentally and numerically [1]. Applying these unit cells in macroscopic components, confirms the importance of the unit cell geometry and highlights the interaction between the scales. An example for this kind of interaction is a more even stress distribution due to the interplay of the auxetic effect of the unit cells and the e. g. a macroscopic encasing. In consequence the stored elastic energy increased compared to non-auxetic unit cells. In order to exploit the full potential of mesostructured components a hierarchic optimisation of spatial distribution, orientation and shape of the unit cells is required. This has been applied to optimize bending dominated structures under static and dynamic Loads [2,3]. Auxetic metamaterials offer a high potential in application but come with drawbacks due to the resolution of the mesostructure alongside with the optimization of large-scale structures. In order to reduce the resulting computational costs, efficient solution algorithms based on reliable error estimates in the linear [4] as well as the nonlinear regime of elasticity.

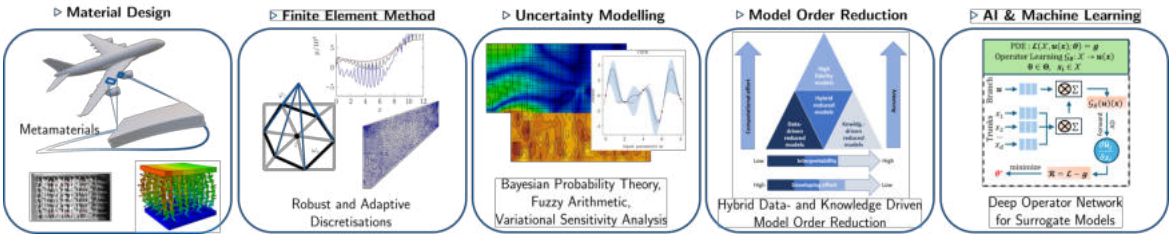

Figure 1. Design concept and methods for auxetic metamaterials.

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Thermomechanical Physics Augmented Neural Networks: Construction, Training and Implementation

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1 Introduction

In this work an overview is given about the construction and use of Physics Augmented Neural Networks (PANNs) with temperature dependent behaviour. The work is based previous works regarding thermoelasticity and neural networks [1, 2] and on the entropic thermoelasticity outlined in [3]. The main novelty of this work is the inclusion of isentropic conditions to more directly couple the different mechanical and thermal responses during training. Network architecture, data generation, training and implementation into a commercial FE software are shown.

2 Neural network training

PANNs have been applied to isotropic, anisotropic, coupled magneto-elastic, or damage effects and many more special cases. PANNs by design include restrictions such as polyconvexity, thermodynamic consistency, non-negativity of the strain energy or some other physically relevant restriction. In this work the framework for modelling all aspects of thermomechanical behaviour that can be derived from the Helmholtz free energy ψ are presented. The general form of the Helmholtz free energy is taken as

$$\psi = \psi_{\mathrm{M}} + \psi_{\mathrm{TM}}\Phi_{\mathrm{T}} + \psi_{\mathrm{TH}}, \tag{1}$$

for the neural network model. It is trained on stress data, as in [4], however in this work isentropic behaviour is assumed during training and additionally the network is trained to predict the specific heat capacity, meaning it is trained on its second derivatives. This means that the loss consists of three parts

$$\mathscr{L} = \mathscr{L}_{\mathbf{S}} + \mathscr{L}_{\eta} + \mathscr{L}_{c}, \tag{2}$$

each corresponding to the stress, isentropic and specific heat capacity loss, respectively. Given that the network consists of 4 smaller networks each modelling an energy from Eq. (1), such isentropic training ensures that all the parts, be they mechanical or thermal, appear at least twice in the loss.

3 Conclusion

The neural network is able to capture the neo-Hookean and Mooney-Rivlin models and the temperature evolution. Also, although trained on isentropic data, the network can be used in a non-isentropic setting.

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Modeling rate-dependent fracture in polymer fiber-reinforced concrete using a phase-field framework

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1 Introduction

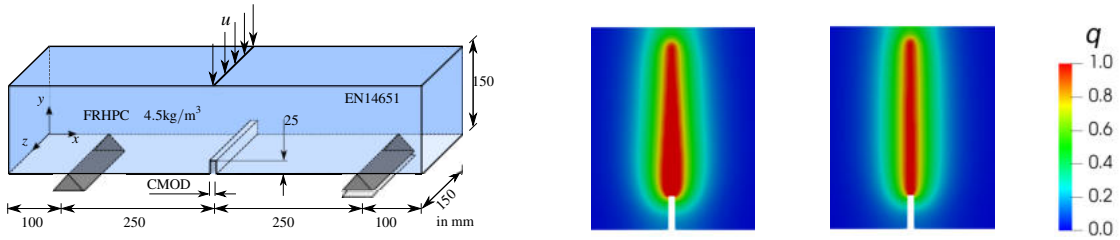
Here, a study is presented, which extends a phase-field model incorporating a transversely isotropic nonlinear viscoelastic Schapery formulation [1] by including orientation distribution functions (ODFs) to investigate the effects of fiber orientation and volume fraction in polymer fiber-reinforced high-performance concrete (HPC). Step-wise linear degradation functions are employed to capture tensile and compressive failure in HPC as introduced in [2]. Various ODFs are implemented to represent different fiber orientations and distributions. Lastly, the performance of the proposed numerical model is assessed based on load–CMOD (crack mouth opening displacement) curves.

2 Numerical framework

The HPC phase is described by a small-strain elastoplastic phase-field fracture model, c.f. [2], while the fibers follow a transversely isotropic nonlinear viscoelastic Schapery formulation, see [1].

Thus, the total energy density function ψ consists of the HPC contribution ψ^{HPC} and the fiber contribution ψ^{F}, weighted by their respective volume fractions $\mathrm{v}^{\mathrm{HPC}}$ and v^{F}. In this regard, the HPC phase depends on the total strain $\boldsymbol{\varepsilon}$, plastic strain $\boldsymbol{\varepsilon}^{\mathrm{p,HPC}}$ and phase-field quantities q and ∇q , while the contribution of the fibers is projected onto the preferred direction using the structural tensor $\mathbf{M}$ and the total strain tensor as given by

$$\psi = \mathrm{v}^{\mathrm{HPC}}\,\psi^{\mathrm{HPC}}\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^{\mathrm{p,HPC}}, q, \nabla q\right) + \mathrm{v}^{\mathrm{F}}\,\psi^{\mathrm{F}}\left(\boldsymbol{\varepsilon}, \mathbf{M}\right). \tag{1}$$



(a) boundary value problem **(b)** $\dot{u} = 1\,\frac{\mathrm{mm}}{\mathrm{s}}$ **(c)** $\dot{u} = 10^{-5}\,\frac{\mathrm{mm}}{\mathrm{s}}$

Figure 1. Finite element analyses of a three-point bending beam test with different loading rate $\dot{u}$.

3 Conclusion

The model presented provides a framework within finite element models which enables the analysis of rate-dependent toughening and crack sensitivity to fiber content and loading rate, cf. Fig. 1.

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Limitations in Fracture Toughness Assessment of Pipes and Improvement of PRNB Testing via an Inclined Notch

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1. Introduction

The Pipe Ring Notched Bend (PRNB) specimen has gained increasing attention as a promising alternative for fracture toughness evaluation of pipe materials, as it preserves the original curvature and stress state of the pipe. However, one of the major limitations of the PRNB specimen is the pronounced non-uniformity of fatigue crack propagation through the wall thickness. Owing to the curved geometry and the specific stress distribution induced by three-point bending, the crack front often develops unevenly, which compromises the accuracy of fracture mechanics parameters and limits the reliability of PRNB specimens compared to standardized specimens.

2. Experimental and numerical research

To address this limitation, a modified PRNB specimen configuration incorporating an inclined through-thickness notch is proposed. The primary objective of this modification is to achieve uniform fatigue crack-front during pre-cracking procedure.
The investigation is based on a combined experimental and numerical approach. Experimental three-point bending tests were conducted on PRNB specimens with both conventional straight notches and the proposed inclined notch configuration. In parallel, detailed finite element analyses were performed to examine the stress distribution, crack-tip fields, and fracture mechanics parameters associated with each configuration. Special attention was given to the evolution of the crack front through the wall thickness during fatigue-precracking procedure.
The results demonstrate that the inclined notch significantly alters the local stress state, leading to a more stable and uniform crack-front development compared to the conventional design.

3. Conclusions

Both experimental observations and numerical results consistently indicate that the inclined-notch PRNB configuration substantially improves crack-front uniformity and stability during fracture testing. This improvement directly enhances the reliability of fracture toughness evaluation and reduces uncertainties associated with uneven crack growth. The proposed modification therefore represents a meaningful advancement in the development of PRNB specimens, bringing their performance closer to that of standard SENB specimens while retaining the advantages of pipe-representative geometry. Overall, the inclined-notch design offers a promising pathway for more accurate and robust fracture toughness assessment of pipe materials.

Acknowledgement

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Computational Modelling of Composite Structures Impact Damage

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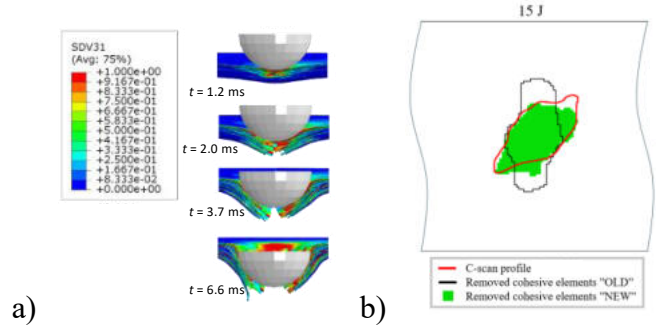
1. Introduction

The work presents an overview of the research activities performed within the research project "Computational Modelling of Composite Structures Impact Damage" financed by the Croatian Science Foundation.

2. Methods and results

The proposed numerical framework is implemented in Abaqus/Explicit using user-defined material subroutines (VUMAT). A mesomechanical finite element approach is adopted, where individual plies are modelled with solid elements, while cohesive elements are used to represent the ply interfaces. A strain-rate dependent damage models for intralaminar ply failure and interlaminar interface failure are combined and further extended through a coupling strategy that enables phenomenological prediction of delamination initiation based on matrix damage in neighbouring plies. In addition, strain-rate effects are introduced into the constitutive response of cohesive interfaces through dynamic modification of interface strength and fracture properties. Some of the results of the numerical simulations are shown in Figure 1.



a) b)

Figure 1. Results of the numerical simulations: a) perforation failure mode [1]; b) delamination damage compared with C-scan results [2]

3. Conclusions

Validation results for low-velocity impact loading show that the proposed approach predicts damage size and orientation more consistently with experimental observations compared to the standard built-in cohesive model. The developed methodology therefore represents a promising step toward more reliable and robust computational approaches for impact damage assessment of advanced composite structures.

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Limitations and Extensions of Eigenerosion Methods for Variational Fracture Modeling

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The prediction of crack initiation and propagation remains a central challenge in computational solid mechanics. Fracture processes introduce displacement discontinuities whose location and evolution are generally unknown a priori (*free boundary problems)*, making their numerical treatment particularly demanding within standard continuum mechanics frameworks. Variational formulations of fracture mechanics, inspired by the classical work of Griffith, have emerged as a powerful theoretical basis for addressing this challenge by recasting crack evolution as an energy minimization problem balancing elastic energy release and surface energy dissipation.

Within this variational framework, a number of computational approaches have been proposed to regularize the free-discontinuity problem. Among the most widely used are phase-field methods, which approximate cracks by a diffuse damage field defined over the domain and require additional degrees of freedom. While these approaches have demonstrated robustness for complex crack patterns, they are computationally very expensive.

An alternative class of methods is based on the concept of eigendeformations. In the eigenfracture framework introduced in [1], cracks are represented through an eigendeformation field that allows displacement discontinuities without local elastic energy cost, while fracture energy is accounted for through a regularized neighborhood of the crack set. This formulation leads naturally to the eigenerosion method, where the eigendeformation field is restricted in a binary sense and crack propagation is represented by the erosion of elements once an energy-based criterion is satisfied. When implemented within the finite element method, eigenerosion offers a conceptually simple and computationally efficient alternative to methods that require additional field variables or enrichment techniques such as phase-field methods, see [2].

While the convergence properties of the eigenerosion method are well established theoretically [1], practical implementations often rely on approximations whose accuracy has not been systematically assessed. For instance, the widely used first-order estimate of the energy release introduced in [2] is commonly adopted due to its computational efficiency, although its accuracy relative to the exact energy balance remains largely unexplored. Moreover, strategies frequently employed in practical simulations – such as eroding multiple elements simultaneously to reduce mesh bias – may not always be consistent with the physical crack evolution, especially in situations where crack growth is stable and proceeds element by element. Finally, the treatment of boundaries introduces additional sources of error: internal crack surfaces and external boundaries may contribute differently to the energy balance, yet they are often handled identically in existing implementations.

The objective of this work is therefore to revisit key numerical ingredients of the eigenerosion approach and to analyze their influence on structural response and crack evolution. Through a series of benchmark examples and systematic numerical studies, we demonstrate that addressing these issues significantly improves the predictive performance of the eigenerosion method while preserving its conceptual simplicity and computational efficiency.

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Simulation of Effective Fracture Parameters in Porous and Hybrid Materials

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1. Introduction

This work investigates the simulation-based determination of the effective fracture resistance of porous and hybrid materials. Using the concept of Hossain et al. [1], numerical experiments to identify effective crack resistance as a material parameter are performed, see [2]. Displacement boundary conditions resulting in a steadily propagating macroscopic crack are applied to a representative porous or hybrid microstructures. At the microscale, crack growth is simulated without prior assumptions on crack path, continuity of propagation, or other features. The maximum value of the macroscopically acting J-integral defines the driving force required to advance the crack by a macroscopic length increment without arrest, which we refer to as the effective crack resistance.

2. Fracture Phase Field

To simulate crack evolution in the microstructure without predefined growth assumptions, we employ a fracture phase field model. Phase field models are particularly well suited for this purpose, as they provide a regularized approximation of cracks and introduce an inherent internal length scale. The length scale in conjunction with the fracture toughness of the material defines a fracture stress. This fracture stress becomes important when crack nucleation at stress concentrators such as holes and inclusions occurs. The facture phase field model is described by a variational approach, allowing for an efficient implementation into the finite element software framework FEniCSx.

3. Conclusions

The simulations reveal that crack re-nucleation is a toughness-determining failure mechanism in porous materials. We investigate the influence of various parameters on the characteristic length at which cracks re-nucleate, revealing a strong correlation with the internal length scale of the phase field model. The effective crack resistance in simplified porous materials, focusing on the effects of pore shape and pore spacing is discussed. Our results highlight their significant influence on crack resistance, providing insights into the failure behavior. If a von Mises plasticity model is employed to investigate ductile fracture in porous metals, the effective fracture resistance is governed primarily by the energy dissipation capacity during crack re-nucleation at pores rather than by tensile strength or microscopic fracture energy.

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Efficient modelling of adhesive debonding: from beam theory to multi-scale interface behaviour

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1 Introduction

The reliable prediction of adhesive joint failure remains a central challenge in structural mechanics, particularly in applications involving mixed-mode loading and complex interface behaviour. While sophisticated finite-element models can provide accurate results, their computational cost often limits their applicability in structural-scale analyses, particularly in situations requiring repeated simulations, such as parameter identification, optimisation and generation of training data for surrogate models.

2 Efficient simulation of adhesive debonding in standard fracture-mechanics tests

This contribution presents a series of computationally efficient and mechanically consistent approaches for modelling adhesive debonding and characterising fracture resistance in beam-like structures. The proposed framework combines analytical solutions and finite-element models based on beam theory capable of simulating complex non-linear behaviour in adhesive debonding. Such a hybrid strategy enables a significant reduction in the number of degrees of freedom while retaining high accuracy in predicting global structural response and local fracture behaviour. The approach is applicable to standard fracture-mechanics tests [1], allowing for robust modelling of damage and crack propagation, as well as characterisation of fracture resistance. Efficient and reliable identification of the traction-separation-law parameters [2] is essential for accurate simulation of non-linear material behaviour in adhesive debonding. The formulation has been successfully extended to account for a rate-dependent response of the adhesive by combining damage and viscosity [3]. In parallel, extensive experimental work has been conducted to validate the developed computational tools and establish reliable measurement and data post-processing procedures [3].

3 Conclusions and future work

Despite these advances, classical interface models often rely on simplified traction-separation laws that do not explicitly account for microstructural effects such as surface roughness, asperity interaction and frictional sliding, which are known to play a significant role in mode-II dominated debonding. To address this limitation, ongoing work is directed towards the incorporation of multi-scale interface descriptions inspired by micro-mechanical approaches, such as multi-plane cohesive zone models that account for asperity geometry and frictional mechanisms at the micro scale. These developments aim to enhance the predictive capability of the framework while preserving its computational efficiency.

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A finite element formulation for stability analysis of open thin-walled composite beam-type structures

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1. Introduction

Since load-bearing composite structures typically feature slender beam elements with thin-walled cross-sections, their response to external loads is significantly more complex. These structures have a higher propensity for stability loss and exhibit a more pronounced risk of buckling and deformation [1]. In the design process, it is crucial to carefully determine the stability limits of deformation forms. Analytical solutions are only applicable in simpler scenarios [1], which makes the development and implementation of numerical methods essential [2,3].

The goal of this study is to perform a nonlinear large displacement analysis for open thin-walled beam-type structures, taking into account shear deformation effects. The cross-section is subdivided into multiple rectangles, with each rectangle consisting of orthotropic layers arranged in a symmetric balanced or unbalanced laminate. The analysis will rely entirely on a numerical model developed by the authors, with the results being compared to those from other relevant sources.

2. Methods

To incorporate shear deformation effects into the formulation, Timoshenko's theory for non-uniform bending and the modified Vlasov's theory for non-uniform or warping torsion are utilized. Additionally, this work presents an enhanced shear-deformable beam formulation that accounts for bending-bending and bending-warping torsion coupling shear deformation effects [2,3]. The beam is assumed to be prismatic and straight, with external loads considered as conservative and static. The element's geometric stiffness is derived using the updated Lagrangian (UL) incremental formulation [5], along with the nonlinear displacement field of a cross-section, which includes second-order displacement terms due to large rotation effects. To address material inhomogeneity in the composite cross-section, a separate numerical model is employed to calculate the cross-sectional properties. These properties are then weighted using three distinct reference moduli, effectively replacing the inhomogeneous cross-section with an equivalent homogeneous one [4]. The three reference moduli used are the longitudinal modulus Q_{11R}, shear modulus Q_{66R}, and coupling modulus Q_{16R}. Each modulus is responsible for handling specific cross-sectional properties.

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Numerical Modelling of LG Structures: Continuum and Discontinuum Approaches under Different Loading Regimes

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1. Introduction

The mechanical response of laminated glass (LG) structures is governed by complex interactions between glass and interlayer combined in a layered cross-section geometry. Polymeric interlayer brings dependence on loading and boundary conditions, while glass creates uncertainty due to various fracture patterns and demands for specific details that are crucial for keeping structures safe. Accurate prediction of performance for these structures requires robust numerical approaches capable of capturing both continuous and discontinuous phenomena. Traditional continuum-based methods, such as the Finite Element Method (FEM), are widely used for structural analysis under static and dynamic loading with moderately nonlinear conditions [1]. Discontinuum-based approaches, such as the Discrete Element Method (DEM) and Combined Finite Discrete Element Method (FEM-DEM), represent a better basis for simulating such problems, especially those involving fragment separation, delamination [2], and large discontinuities. Recent developments focus on combining these approaches to overcome their individual limitations.

2. Numerical Approaches and Modelling Strategies

Numerical models for LG must incorporate several key features to predict structural behaviour accurately. These features can be placed within three structural stages: Ultimate Limit State (ULS), Fracture Limit State (FLS), and Post-Fracture Limit State (PFLS). In ULS, these include a realistic representation of the cross-section (glass and interlayer), interlayer shear deformation, and the effects of time and temperature. In dynamic events of ULS, the models should account for interlayer viscosity and its role in load damping, as well as internal stress distributions for different glass types. Furthermore, in FLS, they must simulate fracture processes, including fragmentation patterns and fragment size, stiffness degradation due to breakage of one or multiple plies [3], and volume expansion in tempered glass upon fracture. Finally, in PFLS, the model should capture mechanisms such as crack-bridging ligaments, fragment interlocking in compression zones, and the hyperelastic behaviour of the interlayer prior to failure.

3. Conclusions

The study outlines the key aspects of numerical modelling of LG structures across different loading regimes, emphasizing the need for appropriate selection of modelling strategies depending on the problem characteristics. While continuum approaches remain efficient for many engineering applications, discontinuum and hybrid methods offer significant advantages in capturing complex failure mechanisms. Ongoing developments in coupled methodologies provide promising directions for future research, particularly in the context of dynamic and impact loading scenarios.

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A Polygonal Finite-Element-Method for the Analysis of Plates and Shells

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In conventional finite element analysis, the parametrization of 2*D* domains relies on a fixed set of element topologies, namely trianglar or quadrilateral element shapes including special variations thereof such as serendipity elements (among others). This poses certain restrictions the tesselation of domains with respect to these elements and in some cases requires user interference. An alternative to these conventional methods are polygonal finite elements that relax the limitations in the meshing process and are especially suitable for versatile mesh generation strategies such as quadtree and Voronoi algorithms. Besides, polygonal meshes offer highly regular meshes and effortless local refinement. Therefore, several polygonal finite element methods have been developed, e.g. Wachspress Finite Elements, Virtual Element Method or the Scaled Boundary Finite Element Method (SBFEM).

In this contribution the SBFEM and its isogeometric counterpart are presented in examples of structural analysis. The method is found to be especially suitable to stabilize mixed finite element formulations [1] or to determine a crack propagation by adaptive refinement of the initial mesh [2]. Further, the superior stress approximation can be utilized in Reissner–Mindlin plate formulation including a locking remedy tailored for polygonal meshes [3]. In curved structures such as shells, the flexibility in the boundary description yields that complex or trimmed domains can be computed in a straight-forward manner by scaled boundary isogeometric analysis as demonstrated in [4], see 1. In conclusion, SBFEM and its isogeometric extension, provide a powerful and flexible framework for the analysis of various structural problems.

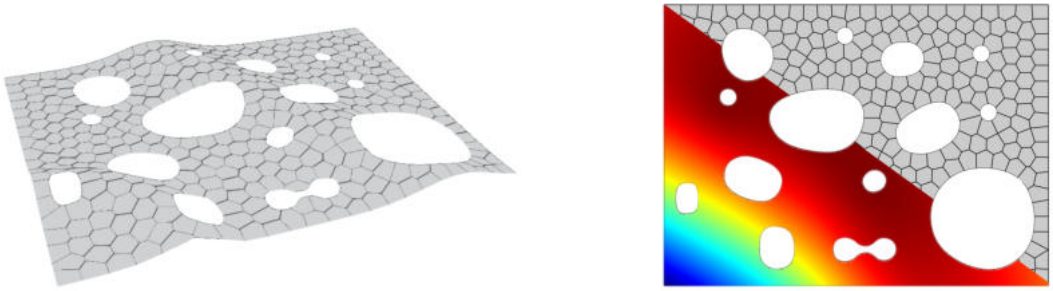

Figure 1. Initial surface of a waved roof structure discretized by polygons inspired by the Learning Center in Lausanne (left) and a divided contour plot of the out-of-plane deformations and its discretization.

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Mixed least-squares finite element formulations for the application in poroelasticity

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We present a mixed least-squares finite element method (LSFEM) for the geometrically nonlinear theory of porous media (TPM) with incompressible solid constituents and viscous pore fluid [1]. The formulation considers a fully saturated binary solid–fluid system, where solid deformation is governed by the balance of linear momentum and fluid transport is described by Darcy's law. This results in a first-order div-grad system that is well suited for least-squares minimization. In the proposed approach, solid stress and deformation as well as pore pressure and fluid velocity are introduced as independent fields. Stress and velocity are discretized using Raviart-Thomas (*RT*) elements, whereas displacement and pressure are approximated by Lagrange polynomials (*P*). Compared with mixed Galerkin methods, the formulation avoids the need to satisfy the inf-sup condition [2] and yields symmetric positive-definite system matrices, including for non-self-adjoint operators. The influence of residual weighting on convergence is studied for higher-order *RT-P* elements using the canonical consolidation problem. A parametric analysis is performed to evaluate the effect of different weights and interpolation combinations on the residual contributions and overall convergence behavior. The results confirm the suitability of the proposed LSFEM for the simulation of incompressible, fluid-saturated porous media.

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Multiscale Analysis of Damage in Thermoplastic Fiber-Reinforced Composites

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1. Introduction

The growing need for lightweight designs in transportation and related industries has led to an increased use of fiber-reinforced thermoplastic composites. This trend is largely motivated by their short processing times, high toughness, and recyclability. At the same time, the macroscopic response of these materials is governed by intricate softening mechanisms driven by damage processes within the thermoplastic matrix. The matrix exhibits rate-dependent visco-elasto-plastic behavior combined with pronounced inelastic deformations, making reliable failure prediction challenging. Current modeling strategies are still limited in this regard, particularly for thermoplastic composite laminates, where multiple interacting mechanisms span different length scales. Moreover, classical multiscale approaches tend to lose objectivity once softening phenomena arise.

2. Methods

In this work, a gradient-enhanced visco-hyperelastic-plastic constitutive model for the matrix is applied within fully resolved composite microstructures. The incorporation of gradient terms eliminates the mesh dependence typically associated with strain-softening, thereby enabling objective predictions at the microscale even under strongly localized deformation.

However, when these localized fields are incorporated into a computational homogenization framework, the resulting effective response exhibits a dependence on the selected representative volume element (RVE) size. This indicates a remaining scale sensitivity that cannot be addressed by gradient regularization alone. To overcome this issue, the microscale behavior is upscaled using a failure zone homogenization approach. This method explicitly considers the finite spatial extent of the failure process during homogenization and reduces the sensitivity of the effective response to the RVE size.

3. Conclusions

By linking mesh-objective gradient regularization at the microscale with a size-objective homogenization strategy at the macroscale, the proposed framework provides a consistent description from matrix-level damage mechanisms to the overall structural response. In doing so, it addresses key limitations of existing approaches and contributes to the development of robust and predictive modeling tools for thermoplastic composite structures.

Acknowledgements

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From global response to strain localization in porous materials: a volumetric perspective

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1. Introduction

Porous materials, particularly cellular metals, combine low density with favourable mechanical properties, making them attractive for lightweight applications. Their behaviour is governed by a complex internal structure. However, the link between global material response and local deformation mechanisms [1], such as strain localization, remains insufficiently understood.

2. Volumetric characterization of porous materials

A key aspect in the characterization of porous materials is capturing their inherently three-dimensional and heterogeneous internal structure. Unlike dense materials, their behaviour is governed by bulk architecture [1], including pore distribution, connectivity, and imperfections, with deformation mechanisms such as cell wall buckling, which leads to strain localization, often developing within the material volume and remaining inaccessible to surface-based techniques [2]. Volumetric approaches, such as X-ray computed tomography (XCT) combined with Digital Volume Correlation (DVC), enable full-field 3D characterization of displacement and strain fields, allowing direct observation of structure–response relationships.
In this work, the transition from global material response to strain localization in porous materials is explored from a volumetric perspective. Using APM foam elements [1] under quasi-static compression, a global (*i.e.*, finite element based) DVC approach [3] applied to in-situ XCT data enables the quantification of 3D strain fields, the identification of localization patterns, and the linking of macroscopic behaviour to local mechanisms.

3. Conclusion

The presented framework provides volumetric insight into the deformation behaviour of porous materials, linking global mechanical response with underlying local mechanisms. It reveals the development of localized phenomena, such as strain concentrations and shear bands, indicating the onset of structural instability, while correlation residuals act as sensitive indicators of damage initiation and provide insight into the underlying damage mechanisms [3]. This perspective enables a more physically meaningful interpretation of deformation processes and highlights the importance of three-dimensional characterization for modelling porous materials.

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Mechanical Characterization of Automotive Materials under Complex Loading Conditions

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1. Introduction

Automotive structures are increasingly designed using lightweight materials in order to reduce vehicle mass, improve energy efficiency, and maintain or enhance passenger safety. However, replacing conventional metallic components with polymer-based or composite materials requires reliable mechanical characterization under complex loading conditions. This work presents an overview of the mechanical characterization of materisls used for door structures, where stiffness, strength, energy absorption, and long-term material behavior are critical design requirements.

2. Material Characterization and FEM model

The work addresses the motivation for replacing conventional metallic components with polymer-based and composite materials, while maintaining stiffness, safety, manufacturability, and cost efficiency [1]. An initial numerical case study compares metallic and plastic door models under selected structural load cases, including door sag, sash rigidity, door over-opening, beltline stiffness, and local attachment loads. Finite element simulations are used to evaluate deformation, equivalent stress, elastic strain, and safety factors, highlighting the importance of model refinement and reliable material data, Fig. 1. Experimental work includes compression testing according to ASTM D695 and creep and stress relaxation testing according to ASTM D2990.

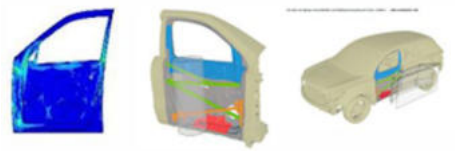

Fig. 1. FEA of front door

3. Conclusions

This work establishes a combined numerical and experimental framework for understanding and predicting the mechanical response of lightweight automotive materials under complex loading conditions, supporting the future development of validated material models.

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Dynamic Behaviour of Rigid-Body Systems with Dry and Elastic Contacts: From Rocking to Ductile Masonry Models

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The dynamic response of block-based structural systems subjected to seismic or other dynamic excitations is nontrivial to describe due to the strongly nonlinear nature of contact interaction, impact, separation, and energy dissipation mechanisms. This challenge motivates ongoing research ranging from the rocking behavior of rigid bodies to the response of masonry systems with deformable and potentially ductile interfaces. In dry-joint systems, such as freestanding block assemblies or masonry structures after bond failure, rocking represents the governing response mechanism. In bonded masonry systems, however, the interaction between blocks is controlled by the constitutive behavior of the joint material, including elasticity, damage evolution, fracture initiation, and crack propagation. The present research focuses on the development of simplified yet mechanically consistent models capable of describing masonry structures with thin deformable interfaces, ranging from elastic response to post-failure rocking and sliding.

A simplified planar numerical model has been developed and shown to provide a satisfactory description of the linear elastic response of masonry walls and masonry arches with thin PU foam joints. Current research extends these models beyond the linear elastic regime through experimental investigations aimed at characterizing the adhesive behavior under different loading conditions and loading rates. Particular attention is given to nonlinear response, damage evolution, and rate-dependent tensile and shear behavior. Complex mechanical response is expected due to the highly porous and heterogeneous microstructure of the PU foams used for this application.

Previous research has shown that masonry structures after bond failure, as well as dry-joint block assemblies, are predominantly governed by sliding and rocking mechanisms. Sliding response is mainly controlled by shear interaction at the contacts, whereas rocking strongly depends on restitution-related energy dissipation during repeated impacts. Our previous investigations demonstrated that many simplified rocking models underestimate energy dissipation because they assume rotation about a single contact point and neglect the distributed nature of impact interaction. Current work therefore focuses on the sliding-rocking response of rigid blocks on deformable supports using a nonsmooth contact dynamics (NSCD) framework, with particular emphasis on reducing sensitivity to temporal and spatial discretizations.

The proposed approach combines cohesive-zone interface models for describing bonded masonry systems prior to failure with nonsmooth contact dynamics formulations used to represent post-failure interaction mechanisms such as separation, impact, sliding, and rocking. This is particularly suitable for masonry systems with modern thin-layer adhesives such as PU foams, whose influence on global dynamic response and damage evolution is still insufficiently understood and represented in simplified dynamic models.

Acknowledgments

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Development of Deep Learning Models for Mapping Infrared Thermography to Full-Field DIC Strain Measurements

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1. Introduction

Digital Image Correlation (DIC) and Infrared Thermography (IRT) are widely used full-field measurement techniques in experimental mechanics [1]. DIC enables non-contact measurement of displacement and strain fields, while IRT allows monitoring of temperature changes during deformation and loading. Their combined use enables simultaneous observation of thermal and mechanical behaviour during experimental testing. However, combined DIC and IRT measurements often require complex experimental setups involving multiple synchronised cameras and demanding data processing, particularly during high-speed testing. Recent developments in deep learning and image-to-image translation methods have shown promising results in computer vision applications [1–3]. Nevertheless, the possibility of directly predicting full-field DIC strain measurements from thermal data has not yet been sufficiently investigated.

2. Methodology and Results

Experimental data were obtained using a servo-hydraulic testing system equipped with a high-speed visible camera and a FLIR infrared camera operating at matched acquisition frequencies. Full-field strain measurements were calculated using the DIC software package. Thermal images and DIC strain fields were spatially aligned and preprocessed to create paired datasets for supervised learning. Several convolutional neural network architectures were investigated, including U-Net encoder–decoder models [2] and Pix2Pix conditional generative adversarial networks [3]. The models were trained to learn relationships between thermal measurements and corresponding strain distributions. Preliminary results demonstrated that the developed models were capable of reproducing the strain localisation patterns from thermal data with satisfactory agreement. The best predictions were obtained in regions characterised by localised deformation and pronounced temperature changes. Evaluation was performed using Root Mean Square Error (RMSE), Structural Similarity Index (SSIM), and Peak Signal-to-Noise Ratio (PSNR).

3. Conclusion

The application of deep learning-based image-to-image translation methods for predicting full-field DIC strain measurements from infrared thermography data was investigated. Preliminary results demonstrated that thermal measurements contain sufficient information for reconstruction of mechanically relevant strain patterns. The proposed approach could contribute to reducing the complexity of experimental setups involving multiple optical systems while enabling simplified thermal–mechanical data fusion for applications in experimental mechanics, structural health monitoring, and dynamic testing.

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Variational Formulations and Computational Frameworks for Chemo-Mechanical Coupling in Solids

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This talk gives an overview of our current research activities in chemo-mechanics-based material modeling. Building on experience in theoretical and computational methods for multiphysics problems [1], our interest lies in modeling nonlinear and dissipative material behavior in which interacting thermo-chemo-mechanical processes play a dominant role. The aim is to capture mechanisms such as stress- and temperature-biased phase transitions and chemical reaction-diffusion processes that directly influence, or even enable, the effective behaviors of modern engineering materials. In this general multiscale approach, model formulations may address very different length and time scales. We consider macroscopic settings that capture the influence of chemo-mechanical coupling in an effective manner, cf. [5], but also phase-field formulations (e.g., of Allen-Cahn and/or Cahn-Hilliard type) that track phase boundaries or crack surfaces in a diffuse interface sense, see [2, 3].
Three fundamental ingredients are addressed: (i) the theoretical model developments, particularly regarding variational settings, (ii) the numerical treatment of these problems, and (iii) the calibration of thermodynamical potentials via the CALPHAD method. Regarding the first aspect, we discuss the advantages and disadvantages of formulations for chemo-mechanical multifield problems through minimization and saddle-point principles. In terms of numerical solution schemes, we elaborate on the finite element implementation of such theoretical frameworks. Our particular approach builds on the flexible and quite general utilization of the UserELement interface (UEL) provided in the FE software package Abaqus. We further discuss ongoing collaborative work on the co-design of the variational model development and parallel solvers for chemo-mechanics problems, for which the MPI-parallel implementation instead is based on the software libraries deal.II, p4est and FROSch (Fast and Robust Overlapping Schwarz), see [4]. Moreover, a concept is proposed in which thermodynamically informed material models are efficiently achieved via CALPHAD-trained neural networks.
Representative numerical examples from a broad spectrum of technologically relevant problems—ranging from hydrogels, multifunctional ceramic filters, to high-temperature electrolyzers for the production of green hydrogen—are presented to demonstrate the robustness, validity and flexibility of our chemo-mechanical simulation frameworks.

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Interpretable to Expressive: Different Flavors of Data-Driven Constitutive Modeling

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1 Introduction

Data-driven constitutive modeling has opened new possibilities for representing complex material behavior with high predictive accuracy. At the same time, major challenges remain regarding interpretability, thermodynamic consistency, and robustness outside the training regime. This talk presents a structured perspective on different flavors of constitutive modeling, ranging from white-box and grey-box neural network models to symbolic approaches.

2 Main Body of the Text

We discuss constitutive artificial neural networks (CANNs) as two complementary model classes. White-box CANNs embed constitutive knowledge directly into the model architecture, leading to transparent and physically interpretable representations [3]. Grey-box CANNs increase flexibility by using neural-network-based constitutive functions while preserving essential physical constraints [4]. Building on this spectrum, we present Constitutive Kolmogorov–Arnold Networks (CKANs), which combine KAN-based learning with symbolification to recover closed-form constitutive relations [1]. For inelasticity, this idea is extended to iCKANs within a thermodynamically consistent framework, enabling the identification of interpretable elastic and inelastic potentials as well as evolution equations [1].

3 Conclusion

Using synthetic and experimental datasets, we demonstrate that white-box, grey-box, and symbolic models each offer distinct advantages in terms of interpretability, flexibility, and predictive performance. Together, these approaches highlight that data-driven constitutive modeling is best understood not as a single methodology, but as a spectrum that bridges physics-based modeling, machine learning, and symbolic model discovery.

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Bayesian Damping-Based Degradation Detection

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1. Introduction

Early detection of degradation or abnormal behaviour is kay for reliable and safety operation of any system. This is based on technical indicators that are directly or indirectly associated with the underlying deterioration processes [1]. In mechanical systems, parameters such as damping or natural frequency are commonly used for this purpose [2] but their estimation during operation is inevitably affected by uncertainty. Consequently, reliable condition assessment requires not only the estimation of the indicators but also a quantification of the corresponding uncertainty, which can be obtained within the Bayesian framework [3].

2. Bayesian analysis for detection of degradation

The core of the Bayesian approach [3] lies in merging the prior distribution $\pi_0(\varphi)$ with the likelihood function $f(x|\varphi)$, providing the posterior distribution $\pi_1(\varphi|x)$, as given in Eq. (1).

$$\pi_1(\varphi|x) = \frac{f(x|\varphi)\,\pi_0(\varphi)}{\int f(x|\varphi)\pi_0(\varphi)d\varphi} \tag{1}$$

For certain combinations of priors and likelihood functions, the posterior distribution has a closed-form analytical solution, known as a conjugate prior. When no such conjugate pair exists, numerical methods such as Markov Chain Monte Carlo (MCMC) must be used.

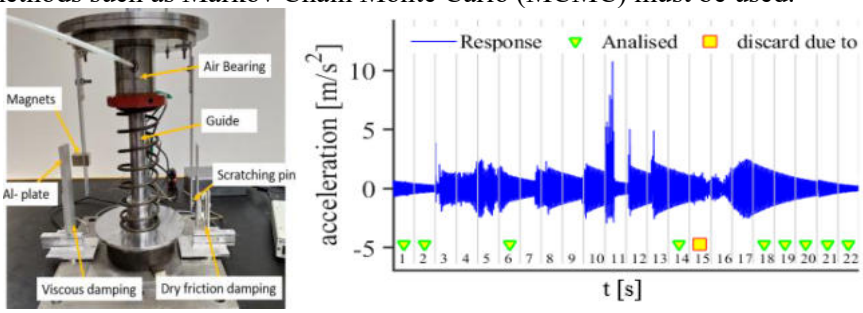

Figure 1. SDoF experimental testbed and analysed signal

To demonstrate the efficiency in estimating damping and its uncertainty, experimental testbeds with SDoF and MDoF systems were used. However, for the sake of brevity, only the SDoF testbed is shown in Fig. 1. The right side of Fig. 1 also shows a typical signal from the SDoF setup.

3. Conclusions

Experiments showed that, beyond the highly efficient estimation of damping, Coulomb friction, and their associated uncertainties, combining Bayesian and Bootstrap methods also enables estimation during operation without load monitoring, making the overall approach highly practical.

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Vascular Biomechanics: Computational Modelling of Diseases and Treatments

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1. Introduction

According to data from the World Health Organization, cardiovascular diseases are the leading cause of death worldwide, responsible for 32% of all deaths globally, of which 85% are caused by heart attacks and strokes. In Europe, this figure is even higher – according to data from 2024, more than 42.5% of all deaths annually are caused by cardiovascular diseases. Currently, there is a significant trend in medicine toward personalized medicine, which has the potential to provide more effective, safer, and more cost-efficient treatments, thereby increasing treatment success rates and reducing the risk of complications. In this context, it is essential to deepen our understanding of cardiovascular diseases and treatment-related complications.

2. Research topics

2.1. Fluid-Structure-Growth model of abdominal aortic aneurysm

Abdominal aortic aneurysm (AAA) is a weakened and bulging region of the aorta that can rupture and lead to life-threatening internal bleeding. Most large AAAs are thrombus-laden, involving a highly complex interplay between aortic wall growth and remodelling (G&R), hemodynamic forces that promote thrombus formation, and biochemical degradation of the aortic wall driven by the proteolytic activity of the thrombus. To describe these processes, a fluid–solid–growth model has been developed. This approach involves a loosely coupled framework that combines computational fluid dynamics simulations over several cardiac cycles (time scale of seconds) with long-term models of wall and thrombus growth and remodelling (time scale of days). Using this model, it is possible to analyse thrombus-laden AAA growth, thrombus deposition, changes in aneurysm shape due to evolving flow fields, and the risk of rupture [1].

2.2. Adaptation of carotid artery to stent implementation and optimisation of stents

Atherosclerosis is a localized inflammatory disease characterized by the buildup of plaque inside of the arteries, that can reduce the diameter of arteries or even cause complete blockage. A common treatment for stenosis is angioplasty with stent implantation. One of the complications is in-stent restenosis (ISR) – gradual re-narrowing of a stented artery due to neointimal tissue proliferation, which is defined as the accumulation of smooth muscle cells and extracellular matrix in the intima. By applying the G&R model to stent implementation, it is possible to simulate arterial adaptation, assess the risk of ISR, and support improvements in long-term treatment outcomes for different stent materials and patient-specific stent geometries.

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Workshop Group Photo